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SCALE-SPACE MUTUAL INFORMATION FOR TEXTURAL-PATTERNS CHARACTERIZATION

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ABSTRACT
The essence of image texture is typically understood by two aspects. First, within a texture-pattern there is a significant variation in intensity values between nearby pixels. Second, texture is a homogenous property at some spatial scale larger than the spatial resolution of the image. Motivated by the essential aspects of image texture, this paper proposes a novel methodology that combines the use of scale-space and mutual information to characterize textural-patterns. Scale-space offers the mechanism for a multi-scale representation of an image, which will be used to address the scale aspect of texture. On the other hand, mutual information provides a measure to quantify the dependency relationship across the scale-space. It has been found that the proposed methodology has the potential to capture different properties of texture such as periodicity, scale, fineness, coarseness, and spatial extent or size. Practical examples are provided to demonstrate the applicability of the proposed methodology.

KEY WORDS
Scale-space, mutual information, texture characterization.

1. Introduction

Despite the lack of a widely agreeable definition of image texture, its essence is typically understood by two aspects. First, within a texture-pattern there is a significant variation in intensity values between nearby pixels. In other words, at the limit of the physical spatial resolution of the imaging system non-homogeneity is the dominant aspect of intensity distribution. Second, texture is a homogenous property at some spatial scale larger than the spatial resolution of the image. Therefore, those two aspects can define a conceptual range for the existence of texture. We do believe very strongly that the core issue of texture understanding and analysis has to do with the spatial scale of analysis. In other words, the textural information cannot be extracted at a single scale. Instead, texture should be viewed as a hierarchical structure that needs to be analyzed at different spatial scales. Motivated by the essential aspects of image texture, this paper proposes a novel methodology that combines the use of scale-space and mutual information to characterize textural-patterns. In particular, scale-space will be used to span the conceptual range of texture and mutual information will be used as a measuring tool across this range. Therefore, the underpinning hypothesis behind the proposed methodology can be abstracted as follows: By subjecting the image to a systematic parameter variations using scale-space, and explicitly measuring the relationship between images under these parameter variations using mutual information, the hierarchical structure of image texture can be made explicit, which remains to be shown. Before diving into this hypothesis, let us review some relevant literature.

Although there has been a huge body of literature that addresses different aspects of texture such as modeling, segmentation, and extraction [1], the reviewed literature in this paper will be concerned with the work that combines the use of scale-space with different measures from information theory. So far there have been very few attempts to combine the scale-space and information theoretic measures for textural analysis. For example, [2] and [3] use the Kullback-Leibler information measure to quantify the distance between images across the scale-space. The Kullback-Leibler information measure expresses the relative entropy distance between two-probability mass functions $p(x)$ and $q(x)$. In other words, the relative entropy, $D(p||q)$, is a measure of the inefficiency of assuming that the distribution is q when the true distribution is p [4]. The Kullback-Leibler measure can be defined mathematically as follows:

$$D(p \parallel q) = \sum_{x \in \Omega} p(x) \log \frac{p(x)}{q(x)} \tag{1}$$

where Ω is an empirical probability space.

It has been shown by [5] that the Kullback-Leibler is not a global measure for the information change in the scale-space at the infinitesimal limit, but this is only of theoretical importance because the scale value is directly related to the size of integration window of convolution, which is typically greater than zero for actual realization. In addition to the previously cited literature, the classical formulation of the entropy and modified ones are used for texture characterization [6, 7].

To conclude our quick introduction, we need to step-back and point to the main idea behind scale-space representation in the context of image features (here: texture). In scale-space fine-textural information is successively suppressed. In other words, textural-elements having a spatial extent less than the scale parameter used in the kernel function that generates the scale-space will vanish. Therefore from an information perspective, the remaining similarity or dependency should be measured in a formal way rather than any thing else and this is exactly the purpose of the mutual information.

This paper is organized as follows. Section 2 explains the basic ideas behind scale-space. Section 3 shows the principles of mutual information. Then section 4 discusses the combined use of scale-space and mutual information. The experimental results and the discussions are shown in section 5. Finally, section 6 concludes the paper.

2. Scale Space

The basic idea of scale-space representation was proposed and formulated independently by Witkin [8] and Koenderink [9]. And the work of Lindeberg [10] and Florack [11] brought a strong focus and more insights to it. Scale-space calls for a multi-scale representation of the image by embedding it into a one-parameter family of derived images; see Figure 1. In simple words, scale-space representation is a framework that allows the analysis of the image at different spatial resolutions. Therefore, the crucial aspect of the scale-space is to let the resolution of an image vary continuously rather than discretely. This has the effect of a gradual suppression of image features as symbolically shown in Figure 1. The name of scale-space is reserved for the convolution of an image function with a Gaussian kernel [10], which typically stated as:

$$g(x, y; \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x^2+y^2)}{2\sigma^2}} \qquad (2)$$

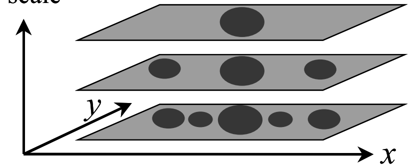


Figure 1: The basic idea of scale-space representation.

σ is the scale factor. The original resolution of the image is indicated by setting σ=0. Thus *g(x,y;0)* is the image function at the original resolution.

3. Mutual Information

The desire for a measure of information of a message came from communication theory [4]. The most commonly used measure of information in signal and image processing is the Shannon-Wiener entropy measure *H* [12]. *H* is typically defined as:

$$H = -\sum_i P_i \log p_i \qquad (3)$$

H is the average information supplied by a set of *i* symbols whose probabilities are given by $p_1, p_2, p_3, .., p_i$. The entropy can be interpreted in three ways: The amount of information an event gives when it takes place, the uncertainty about the outcome of an event, and the dispersion of the probabilities with which the events takes place. The information measure of concern in this paper has to do with the joint entropy with respect to two marginal entropies *A* and *B*. This measure was also introduced by [12] as "rate of transmission of information" and has become known as the mutual information *I(A,B)*, which typically defined as:

$$I(A,B) = \sum_a \sum_b P_{AB}(a,b) \log \frac{P_{AB}(a,b)}{P_A(a) P_B(b)} \qquad (4)$$

$P_{AB}(a,b)$ is the joint probability of two sources *A* and *B* (here: two images). A more compact representation of equation (4) can be written as follows:

$$I(A,B) = H(A) + H(B) - H(A,B) \qquad (5)$$

H(A) is the marginal entropy of *A*, *H(B)* is the marginal entropy of *B*, and *H(A,B)* is the joint entropy of *A* and *B*. Equation (4) or (5) can be interpreted as the distance between the joint distribution of two sources (here: A and B) and the joint distribution in case of independence of them. In other words, it is a measure of dependence between two sources. A useful way of visualizing the relationship between these entropies can be shown by Venn diagrams; see Figures 2, 3, and 4.

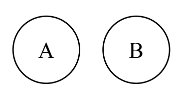


Figure 2: Marginal Entropies.

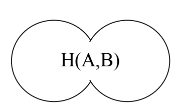


Figure 3: Joint Entropy.

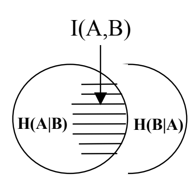


Figure 4: Mutual Information.

Mutual information has the following properties [13]:

a. *I(A,B)=I(B,A)*. It is symmetric; otherwise it would not be mutual information.

b. *I(A,A)=H(A)*. The mutual information of a single source is equal to its entropy.
c. *I(A,B)* ≤ *H(A), I(A,B)* ≤ *H(B)*. The information contained by the sources about each other can never be greater than the information in the sources themselves.
d. *I(A,B)* ≥ *0*. The uncertainty about source *A* cannot be increased by learning about source *B*.
e. *I(A,B)=0* if and only if *A* and *B* are independent.

4. Scale-Space Mutual Information

Scale-space mutual information methodology is about measuring the mutual information between images across scale-space. For example, the mutual information of image A at two different but consecutive scales, σ_i and σ_j, is:

$$I(A_{\sigma_i}, A_{\sigma_j}) = H(A_{\sigma_i}) + H(A_{\sigma_j}) - H(A_{\sigma_i}, A_{\sigma_j}) \qquad (6)$$

Equation (6) will be used to track the relative dependency or the mutual information between consecutive images across the scale-space. Now the question is: What is the meaning of equation (6) with respect to textural information? Before answering this question, let us assume that $\sigma_i < \sigma_j$. This implies that the textural information that has a spatial extent $\leq \sigma_i$ will be suppressed. And similarly, any textural information that has a spatial extent $\leq \sigma_j$ will also be suppressed, which will automatically suppress the textural information at σ_i. Therefore, equation (6) is about the similar textural information, between two images, that has a spatial extent greater than σ_i and σ_j. This fact can be easily seen in the light of Figure 4, which shows the mutual information as an intersection of two sets.

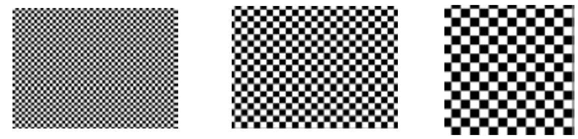
Equation (6) depicts the mutual information between two images in a relative mode, but this equation can also be formulated in an absolute mode by relating all of the images to a single image. Thus, the absolute mode of this equation is realized by relating all of the images across the scale space to the image at the original resolution, which can be stated mathematically as follows:

$$I(A_0, A_{\sigma_j}) = H(A_0) + H(A_{\sigma_j}) - H(A_0, A_{\sigma_j}) \qquad (7)$$

A_0 is the image at the original resolution. Once again, equation (7) is about the similar or mutual information that has a spatial extent greater than or equal to σ_j.

5. Experimental Results and Discussions

Several experiments will be carried out to show the different aspects of scale-space mutual information. The first three experiments will be used to show the usefulness of the proposed methodology to characterize texture periodicity and its spatial extent or scale. Equation (6) will be used first. Three checkboard patterns with different pixel sizes are used in these experiments as shown in Figure 5. Figure 5.a has a pixel size of 1 × 1, Figure 5.b has a pixel size of 2 × 2, and Figure 5.c 4 × 4.



a. b c.

Figure 5: Three Checkboard Texture Patterns.

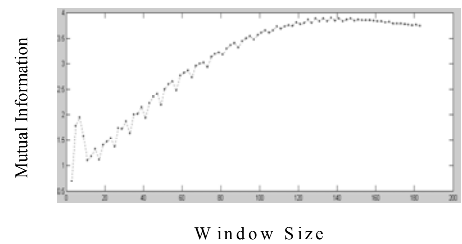


Figure 6: The Scale-Space Mutual Information for Figure 5.a.

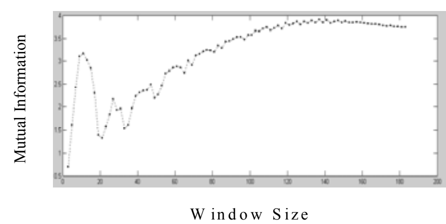


Figure 7: The Scale-Space Mutual Information for Figure 5.b.

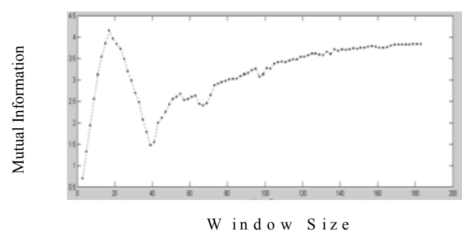


Figure 8: The Scale-Space Mutual Information for Figure 5.c.

By analyzing the information content of the three graphs shown in Figures 6, 7, and 8, it is very evident that the scale-space mutual information captures the periodicity and the scale aspect of the checkboard texture patterns. This is manifested by the short, medium, and long cycles in the three graphs respectively. In other words, fine, medium, and coarse texture can be detected. In addition, those three graphs show that the checkboard texture has a long lifetime over the scale-space. In other words, there is a remaining clue about the texture identity (here: checkboard) in the very large convolution window. Figure 9 supports this claim in which a convolution window of a size of 121 × 121 is applied to the texture shown in Figure 5.a. It is very important to remember that the convolution window size, *w*, is directly related to Gaussian scale, σ, by the following equation:

$$w = 2\sqrt{2}\sigma \tag{8}$$

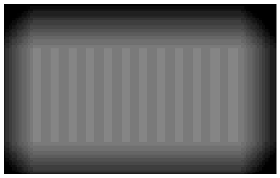


Figure 9: The Gaussian Image, generated by w= 121 × 121, for the Texture shown in Figure 5.a.

Figure 10 shows another example of a periodic texture of a brick wall. Again, this example shows the periodicity of the texture as depicted by the mutual information graph. This periodicity occurs over very large intervals because the size of the texture elements is bigger than the checkboard textures shown previously.

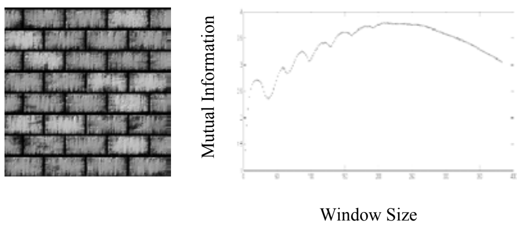


Figure 10: A Brick Wall Periodic Texture.

Now, let us examine equation (7) on Figure 5.a. This equation depicts the absolute mode of mutual information. The result of this examination is shown in Figure 11. It is very evident that the periodicity is captured at small windows or scales but not with the level of details provided by the relative mutual information depicted by equation (6). Moreover, this periodicity is highly suppressed at large scales. This is not a surprise at all because the information distance, so to speak, is very long in the absolute mode. On the contrary, the relative mode of the mutual information preserves the details at small and large scales. Again, this is also not a surprise because in the relative mode more information are kept and the mutual information captures the promise of the continuous representation offered by scale-space.

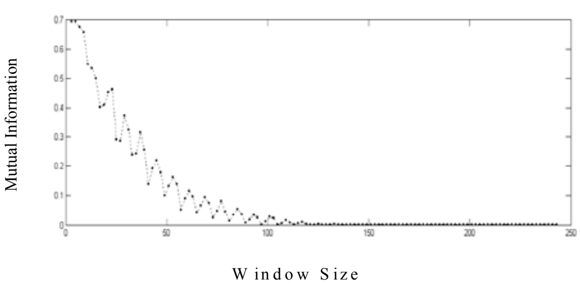


Figure 11: An Example for the Relative Mode of Mutual Information.

The next experiment will demonstrate that the scale-space mutual information in its relative mode could serve as an effective size-measuring tool for texture elements. A snakeskin texture pattern will be used in this experiment; see Figure 12. A zoom-in is applied to the left part of Figure 12 and is indicated by a red square. The result of the zoom-in is shown to the right. The red double-head arrows in the right part of Figure 12 point to the diameters of different texture elements of the snakeskin. These diameters are measured manually and they are found to have an average length of 9.5 ± 0.5 pixels. Now the task is to show that this average diameter could be inferred from the scale-space mutual information graph or function shown in Figure 13.

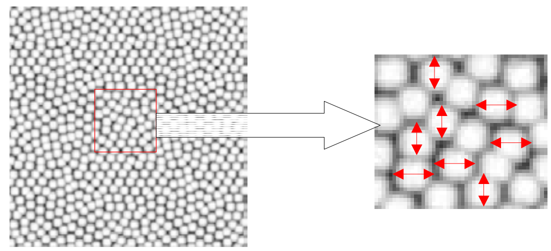


Figure 12: Snakeskin Texture Pattern.

A careful examination of Figure 13 reveals the following. First, the zero window size points to the image at its original resolution and the corresponding mutual information points to the entropy, which can be viewed as a self-mutual information according the properties stated in section 3. Second, the points of inflections of the scale-space mutual information function carry a great deal of information about the characteristics of the texture pattern. For example, point A at Figure 13 shows a rapid drop in the mutual information compared to the entropy or self-mutual information at the original resolution (5 bits to 2 bits). We see this characteristic as a very revealing evidence for the understanding of texture essence. The most important event happened at point B in which the mutual information drops to its lowest value and at the window size of 13×13. According to equation (8), the effective diameter at this window size is equal to 9.19 pixels. It is not difficult to infer that this diameter corresponds to the diameter of the dominant coarse texture elements measured manually. This diameter is within the uncertainty range estimated by the manual measurements. Now the question is: How do we come up with this inference? Scale-space theory gives us a straightforward answer to explain this inference. At this window size the effective diameter is equal to or greater than the spatial extent of the texture elements. Therefore, those texture elements are suppressed. This suppression made the dependency or the mutual information between the smoothed image at this window (13×13) and the previous one (window size: 11×11) very small. Figure 14 shows the original image convolved by two different window sizes (11×11 & 13×13). The small window size is associated with left image and the large one is associated with the right image. This is an additional visual support to show and explain the information suppression at the widow size of 13×13, which corresponds to the dominant texture elements. Beside the dominant texture elements measurement, this experiment also demonstrates that the window size of 13×13 is the optimal one for this image of snakeskin texture.

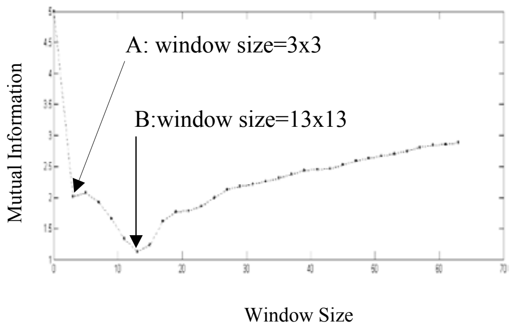


Figure 13: Scale-Space Mutual Information of the Snakeskin Textural Pattern.

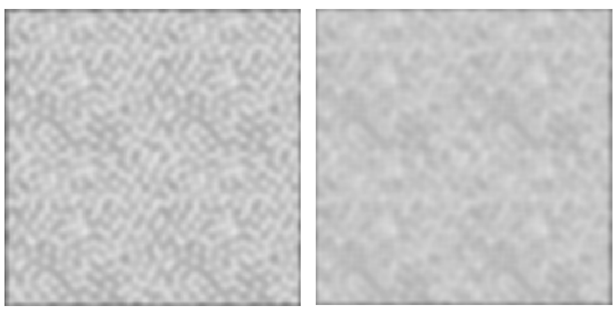


Figure 14: Two Snakeskin Images Convolved With Two different Window Sizes (11×11 & 13×13).

The following experiment will show the use of scale-space mutual function to characterize fine-texture of a grass image.

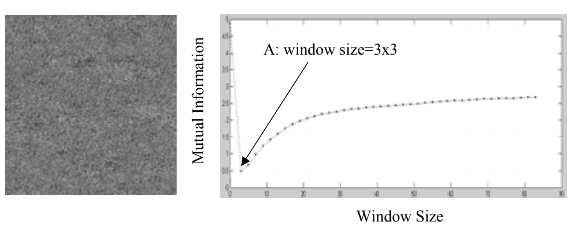


Figure 15: A Grass Image with its Scale-Space Mutual Information Function.

It is very clear from Figure 15 that the grass texture is very fine according to scale-space mutual information and the visual confirmation. Also, it is very interesting to note that there is no textural information beyond a 3×3 window since the graph starts to move to a gradual increased in the relative dependency between successive images across the scale-space. In other words, the major dominant inflection point happened at a 3×3 window. Therefore, it is very evident to state that this textural pattern can be characterized very objectively at a very small scale.

The next experiment shows the use of the scale-space mutual information in a relatively coarse soil texture. Figure 16 indicates that the mutual information reached its lowest value at a 9×9 window. By the same token used in the previous experiments, this window corresponds to the dominant coarse texture.

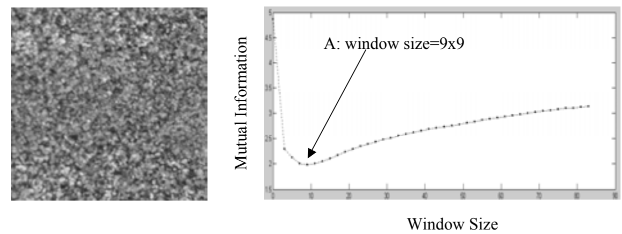


Figure 16: A Soil Image with its Scale-Space Mutual Information Function.

The last experiment demonstrates the use of the scale-space mutual information on a land cover vegetation image acquired by airborne camera. As shown in Figure 17, the dominant coarse texture occurs at a 5×5 window.

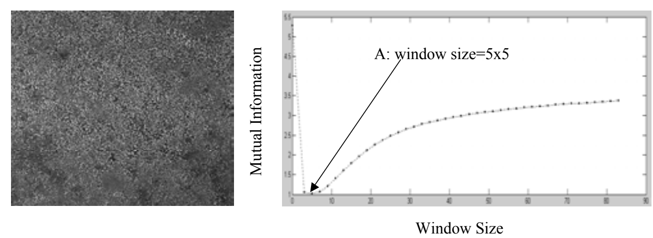


Figure 17: A Vegetation Image with its Scale-Space Mutual Information Function.

6. Conclusion

In this paper we present preliminary results of a new methodology for texture characterization. This methodology is grounded on scale-space representation and mutual information tracking across the scale-space. Scale-space mutual information seeks to explain and quantify the essence of texture as occurred at different spatial scales. Thus, this methodology should not be viewed as a coarse-to-fine strategy. It searches for textural information across a range of spatial scales. It has been shown that this methodology has the potential to capture different properties of texture such as periodicity, scale, fineness, coarseness, and even the spatial extent or size of the texture element.

Indeed, more work need to be done to understand the full potential of this methodology and its relationship to the existing approaches for texture characterization.

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